Uncertainty, calibration and validation of the Mississippi Irrigation Scheduling Tool model

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Implementation and use of a model requires an estimate of its accuracy. The Mississippi Irrigation Scheduling Tool (MIST) is an on-farm decision support tool to assist farmers in irrigating. The accuracy of the model is critical in designing good water management protocols. This research presents the results of the uncertainty analysis of the MIST model, showing the margin of error (uncertainty) of the irrigation advice. The basis for the verification and validation of the model is also given. The MIST calculates the daily soil water balance in a crop field from daily weather measurements, irrigation, and rainfall, accounting for crop type, planting date, soil type, tillage, and other field-specific information. The model output informs farmers of when irrigation is needed. The uncertainty analysis determines the margin of error in the irrigation decision and gives a range within which irrigation is feasible. The current uncertainty analysis also gives essential information on the influence of input parameters on the final irrigation recommendation calculated by the water balance.

The uncertainty calculations were based on Taylor's Series Method for the calculation of the total systematic uncertainty arising from measurement error of variables in the water balance calculation. The errors in measurement were one standard deviation in range, equivalent to an uncertainty with a confidence level of 68.2%. Because the current day's soil water balance depends on the previous day's water balance, the computations are iterative. As equations cascade to calculate the daily water balance, the uncertainties also propagate through the equations. Initially, uncertainty quantifications were performed for two sets of water balance calculations using local weather data. The final uncertainties for the water balance were of the order 3-6%, which is within the acceptable range for error.

The MIST water balance calculations were validated using local weather data consisting of rain days, and significant changes in the solar radiation, relative humidity and wind speed. The final water balance results showed values within acceptable ranges and comparable to in situ measurements of soil moisture. The final relative uncertainty in the water balance value was around 9%, which is in the normal range of margin of error. The current MIST web-based application and uncertainty quantification have been verified and validated for current parameters. The accuracy of the model was shown to be suitable for use by farmers in the Mississippi Delta area, and will help improve water management in crop production systems.

INTRODUCTION

Experimentation has been an essential attribute of humankind. We are all familiar with iconic lab coat-clad scientists in television advertisements presenting results that seem accurate and convincing to the audience, who are drawn to buy a commodity. The accuracy and degree of goodness of such data is occasionally questioned and seldom investigated. As an engineer or scientist, one realizes that all experimental data are subject to interpretation and include a certain amount of inaccuracy or uncertainty. To improve confidence in the accuracy of measured or estimated data to the true value, researchers have been driven to seek methodologies to quantify the errors. Quantification of uncertainties sheds light on the validity and limitation of the data. Therefore, uncertainty analysis of experimentally measured data and modeled results presents a formal methodology to quantify the errors arising from measuring and interpreting data.

In the late 1970's, the lack of international consensus among various scientific societies and authorities on uncertainty in experimental and computational measurements prompted Comite International des Poidset Mesures (CIPM) to establish international guidelines for the methodology of uncertainty analysis. This led to the International Organization for Standardization (ISO) setting up a task force to develop a guideline document. The product of the ISO task force was the "Guide to the Expression of Uncertainty in Measurement", also known as GUM (Bipm et al. 1993), which is now the international standard for the expression of uncertainty in measurements. Later, the North Atlantic Treaty Organization (NATO) Advisory Group for Aerospace Research and Development (AGARD) came up with a quality assessment methodology, through uncertainty analysis, for wind tunnel testing data (AGARD D-AR-304, 1994). With minor revisions in AGARD D-AR-304 (1994), American Institute of Aeronautics and Astronautics (AIAA) published its report on uncertainty analysis (AIAA Standard, 1995). The above mentioned reports comprise the set of standards for uncertainty analysis in engineering. Based on the recommendations of GUM and AIAA Standard S-071-1995 (1995), Coleman and Steele (1995) developed a less complex "large sample" uncertainty methodology. Coleman and Steele (1995) were able to show that their assumptions were less restrictive in the formulation of uncertainty propagation. GUM (1993) and Coleman and Steele (1995; 2009) encouraged researchers to distinguish uncertainties into those that are caused by variability and those that are not, which can be broadly distinguished as random and systematic uncertainty.

The recent development in uncertainty analysis in the engineering field has enabled scientists and engineers to combine and propagate uncertainties from experiments into the modeling stage. To improve its applicability to crop production, the implementation of the Mississippi Irrigation Scheduling Tool (MIST) and use of the web-based model requires an estimate of the accuracy of the simulation results. The following sections describe the uncertainty analysis methodology and the results and discussion deduced from the uncertainty analysis of the MIST web-based application.

UNCERTAINTY METHODOLOGY

Uncertainties arise in a measured variable through a vast number of sources such as an imperfect instrument calibration process, standards used for calibration, influence on the measured variable due to inconsistencies in ambient temperature, pressure, humidity and vibrations. Furthermore, uncertainties are also results of unsteadiness in a "steady-state" process being measured and undesirable interactions between the transducers and environment. In essence, the uncertainty that arises due to variability or randomness of a measured quantity (in this case, soil water balance on a given day: w_t) is referred to as random standard uncertainty (s_{w_t}) and uncertainties that do not arise from variability, but are calculated either through TSM or Monte-Carlo Method (MCM), are called systematic standard uncertainty (b_{w_t}). The total experimental uncertainty (U_{w_t}) is then calculated through a root sum square method specified by the following equation:

$$U_{w_t} = \sqrt{{s_{w_t}}^2 + {b_{w_t}}^2} \quad (1)$$

The level of confidence of the uncertainty is 68%, meaning that the true value of w_t, at a given time, is expected to lie within the bounds of $\pm U_{w_t}$ 68% of the time.

The uncertainty in the result is given by the following expression:

$$U_r^2 = (\frac{\partial r}{\partial X_i})^2 U_{X_1}^2 + (\frac{\partial r}{\partial X_2})^2 U_{X_2}^2 + \cdots (\frac{\partial r}{\partial X_J})^2 U_{X_J}^2 \quad (2)$$

where U_x are the uncertainties in the measured variables X_i. The measured values of X_i are independent of another, and the uncertainties in the measured variables are also independent.

By dividing each term in the equation by r^2, the following equation is obtained from Eq. (2).

$$\frac{U_r^2}{r^2} = \left(\frac{X_1}{r}\frac{\partial r}{\partial X_1}\right)^2\left(\frac{U_{X_1}}{X_1}\right)^2 + \left(\frac{X_2}{r}\frac{\partial r}{\partial X_2}\right)^2\left(\frac{U_{X_2}}{X_2}\right)^2 + \cdots \left(\frac{X_J}{r}\frac{\partial r}{\partial X_J}\right)^2\left(\frac{U_{X_J}}{X_J}\right)^2 \quad (3)$$

where $\frac{U_r}{r}$ the relative uncertainty, and the factors $\frac{U_x}{xi}$ are the relative uncertainties for each variable. The factors which multiply the relative uncertainties of the variables are uncertainty magnification factors (UMF), and defined as:

$$UMF_i = \frac{X_i}{r}\frac{\partial r}{\partial X_i} \quad (4)$$

The relative uncertainty is increased by a UMF less than 1, and the relative uncertainty is decreased if the value of the UMF is greater than 1.

RANDOM UNCERTAINTY

Random uncertainties occur as a result of precision limitations and the inability to replicate data from test to test. Randomness of experimental data is noticed by its scatter or spread in relation to the measured variable. Standard deviation (σ) of data gives an estimate of the extent of the spread. Although there are many key factors that help determine the random uncertainty, repetitive temperature and water balance measurements on the field are the prime reasons of limitations when attempting experimental duplication. The value for σ is calculated from the following expression:

$$\sigma = \sum_{i=1}^{N}\sqrt{\frac{1}{N-1}(x_i - \bar{x})^2} = 0.5 \cdot s_{\sigma_t} \quad (5)$$

where $\bar{x}$ is the arithmetic mean of N tests and x_i is the test data for the ith repetition. The resulting bounds of the random uncertainty bands give a 68% confidence interval. The assessment of the random uncertainty requires substantial experimentation, and as such will be part of the analysis in the next stage of the project.

SYSTEMATIC UNCERTAINTY

The systematic uncertainty can include calibration, data acquisition, data reduction, or conceptual errors. But unlike random error, systematic error is based solely upon inaccuracies in measurement. These measurements have an associated offset, such that each measurement provided by the system contains a degree of inaccuracy. Therefore upon calculation, the systematic error is determined to be a quantity or component of the total error that remains constant at any given time. The systematic uncertainty in the MIST modeling procedure is due to the margin of error in data measurement (strain gage, digital calipers and digital weighing scale). The propagation of systematic error in measuring the water balance through Equation 1 is given by:

$$b_{w_t} = \left\{\left(\frac{\partial w_t(t)}{\partial w_{t-1}}\right)^2 \cdot b_{w_{t-1}}^2 + \left(\frac{\partial w_t(t)}{\partial ET_t(t)}\right)^2 \cdot b_{ET_t(t)}^2 + \left(\frac{\partial w_t(t)}{\partial K_c}\right)^2 \cdot b_{K_c}^2 + \left(\frac{\partial w_t(t)}{\partial RO(t)}\right)^2 \cdot b_{RO(t)}^2\right\}^{1/2} \quad (6)$$

where b_{w_t} is the systematic uncertainty of the final water balance that has to be calculated; $b_{w_{t-1}}$ and $b_{ET_t}(t)$ are the systematic errors in the measurement of the previous day's water balance and evapotranspiration equation respectively; b_{K_c} and $b_{RO(t)}$ are the uncertainty in the calculation of the crop coefficient and runoff respectively (which are essentially the same). The list of the Uncertainty Magnification Factors (UMFs) associated with each parameter of the MIST tool are given in Table 1.

RESULTS AND DISCUSSION

The uncertainty calculations were based on Taylor's Series Method (TSM) for the calculation of the total systematic uncertainty arising from error in the measurement of each variable associated with the water balance calculation. The errors in measurement were one standard deviation in range, which amounts to an uncertainty with a confidence level or accuracy of 68.2%. During the course of calculation the systematic uncertainty of a measured variable does not change. For instance, the temperature measured in the field may have an error of ±0.5 °F, which normally amounts to 0.5-2% relative error; this will remain a consistent error throughout the calculations. This relative error then becomes the relative systematic uncertainty that was used in the uncertainty quantification of the MIST web-based application. Because the current day's soil water balance depends on the previous day's water balance, the computations are iterative. As equations were cascaded to calculate the daily water balance, the uncertainties were also propagated through the same set of equations. Initially, uncertainty quantifications were performed for two sets of water balance calculations based on local weather data obtained from the Mississippi Delta, spanning separate 7-day periods. The final uncertainties for the water balance were of the order 3-6%, which is within the acceptable range for error involved in such calculations. These test cases then become the verification process for the MIST web-application implementation and its uncertainty quantification.

The MIST water balance calculations were validated using the local weather data that consisted of a few days of rain, and significant changes in the solar radiation, relative humidity and wind speed. The result of the analysis on the verification of the MIST tool showed that the relative uncertainty of the water balance varies from -4.5 % to 4.5 % (Figure 1). Further, the rainfall affects the uncertainties significantly for the day having rainfall and the day after rainfall.

CONCLUSIONS

The final water balance results showed values within acceptable ranges and comparable to in situ measurements of soil moisture. The final relative uncertainty in the water balance value was around 9%, which is in the normal range of the margin of error. The current MIST web-based application and uncertainty quantification have been verified and validated with limited preliminary data. Further verification and validation is needed for the entire range of Mississippi weather and irrigation.

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Table 1. The UMFS and corresponding mathematical expressions.

No.	From which eqn.	UMF Mathematical Term
UMF1	$\frac{b_{R_{NS}}}{R_{NS}}$	1
UMF2	$\frac{b_{R_N}}{R_N}$	$\frac{R_{NS}}{R_{NS}-R_{NL}}$
UMF3	$\frac{b_{R_N}}{R_N}$	$-\frac{R_{NL}}{R_{NS}-R_{NL}}$
UMF4	$\frac{b_{R_{NL}}}{R_{NL}}$	$\frac{4T_{max}^4+T_{min}^4T_{max}}{T_{max}^4+T_{min}^4}$
UMF5	$\frac{b_{R_{NL}}}{R_{NL}}$	$\frac{4T_{max}^4+T_{min}^4T_{max}}{T_{max}^4+T_{min}^4}$
UMF6	$\frac{b_{R_{NL}}}{R_{NL}}$	$\frac{0.34-\frac{0.14}{2\sqrt{e_a}}}{0.34-0.14\sqrt{e_a}}e_a$
UMF7	$\frac{b_{R_{NL}}}{R_{NL}}$	$\frac{1.35R_s}{1.35R_s-0.35R_{so}}$
UMF8	$\frac{b_{R_{NL}}}{R_{NL}}$	$-\frac{1.35R_s}{(1.35R_s-0.35R_{so})}$
UMF9	$\frac{b_{ET}}{ET}$	$\frac{0.408\Delta R_n}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF10	$\frac{b_{ET}}{ET}$	$\frac{0.408\Delta R_n}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF11	$\frac{b_{ET}}{ET}$	$\frac{0.408\Delta R_n}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF12	$\frac{b_{ET}}{ET}$	$\frac{0.408\Delta R_n}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF13	$\frac{b_{ET}}{ET}$	$\frac{\frac{u_2 900\gamma e_s}{T+273}}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF14	$\frac{b_{ET}}{ET}$	$\frac{-\frac{u_2 900\gamma e_a}{T+273}}{0.408\Delta(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)}$
UMF15	$\frac{b_{ET}}{ET}$	$\frac{\Delta 0.408(R_n-G)(\Delta+\gamma(1+0.34u_2))+[(0.408\Delta(R_n-G)+\frac{u_2 900\gamma(e_s-e_a)}{T+273}}{(\Delta+\gamma(1+0.34u_2))\{0.408(R_n-G)+\gamma\frac{900}{T+273}U_2(e_s-e_a)\}}$
UMF16	$\frac{b_{u_2}}{u_2}$	1
UMF17	$\frac{b_{u_2}}{u_2}$	$\frac{67.8Z}{\ln(67.8Z-5.42)(67.8Z-5.42}$
UMF18	$\frac{b_{e_s}}{e_s}$	$\frac{e^0(T_{max})^2}{2(e_s)}$
UMF19	$\frac{b_{e_s}}{e_s}$	$\frac{e^0(T_{max})^2}{2(e_s)}$
UMF20	$\frac{b_{e^0(T)}}{e^0(T)}$	$\frac{T(17.27-17.27T)}{(T+237.3)^2}$
UMF21	$\frac{b_{e_a}}{e_a}$	1
UMF22	$\frac{b_{e_s}}{e_s}$	1
UMF23	$\frac{b_{e^0(T)}}{e^0(T)}$	$\frac{e^0(T)}{RH}$
UMF24	$\frac{b_\Delta}{\Delta}$	$\frac{(17.27-17.27T-2(T+237.3)^3)T}{(T+237.3)^4}$
UMF25	$\frac{b_{R_{So}}}{R_{so}}$	$\frac{R_a 2\times 10^{-5}\times Station\ elevation}{R_{so}}$
UMF26	$\frac{b_{R_{So}}}{R_{so}}$	$\frac{R_a 2\times 10^{-5}\times Station\ elevation}{R_{so}}$

Figure 1. The relative uncertainty of water balance during a week from 3/23/2012 to 3/29/2012.