THE USE OF DIGITAL MODELS FOR EVALUATING THE EFFECTS OF DEWATERING IN THE TENNESSEE-TOMBIGBEE DIVIDE CUT AREA, MISSISSIPPI

by

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INTRODUCTION

Digital modeling techniques have become widely used in solving problems in all phases of hydrology. Ground-water flow models are now used extensively to predict responses of systems to stresses.

Modeling techniques are commonly used to simulate local or regional ground-water flow and to aid in the management of ground-water resources. In addition to these applications, the usefulness of digital modeling techniques for engineering and construction purposes is becoming increasingly apparent.

Previous solutions to ground-water flow problems were obtained from analytical solutions to flow equations. These solutions were applicable if simplified and idealized aquifer systems and stress conditions were being considered. With digital modeling techniques, however, problems involving more complex geometry and flow characteristics can be solved.

The Mississippi District of the U.S. Geological Survey studied three ground-water flow problems related to dewatering in cooperation with the Nashville District, U.S. Army Corps of Engineers. The study involved the evaluation of three proposed methods of head reduction for the construction of the Divide Section of the Tennessee-Tombigbee Waterway. Two of these involved actual dewatering while one involved artesian pressure relief.

In the Divide Section, the Waterway will be constructed in Upper Cretaceous aquifers (fig. 1). The Eutaw Formation, the uppermost of these aquifers, consists of discontinuous beds of sand and clay. Generally, the Eutaw yields small amounts of water to wells in the area to be dewatered. Pumping tests conducted in the area indicate a lack of areal uniformity in rates of vertical flow.

Underlying the Eutaw Formation is the McShan Formation. The McShan consists of thin-bedded sand and sandy clay, and is generally a confining or semi-confining layer between the Eutaw Formation and the underlying Gordo Formation. Aquifer tests indicate that the McShan is relatively impermeable in places and is somewhat permeable in other places. The natural pressure head in all three formations is as much as 150 ft above the base of the cut in this area. In places where the Waterway will be excavated down to or near the top of the Gordo Formation there will be a danger of the high head in the Gordo causing "blowouts" in the bottom of the Waterway cut. In anticipation of this problem, the possibility of installing free-flowing pressure relief wells in the Gordo, was studied.

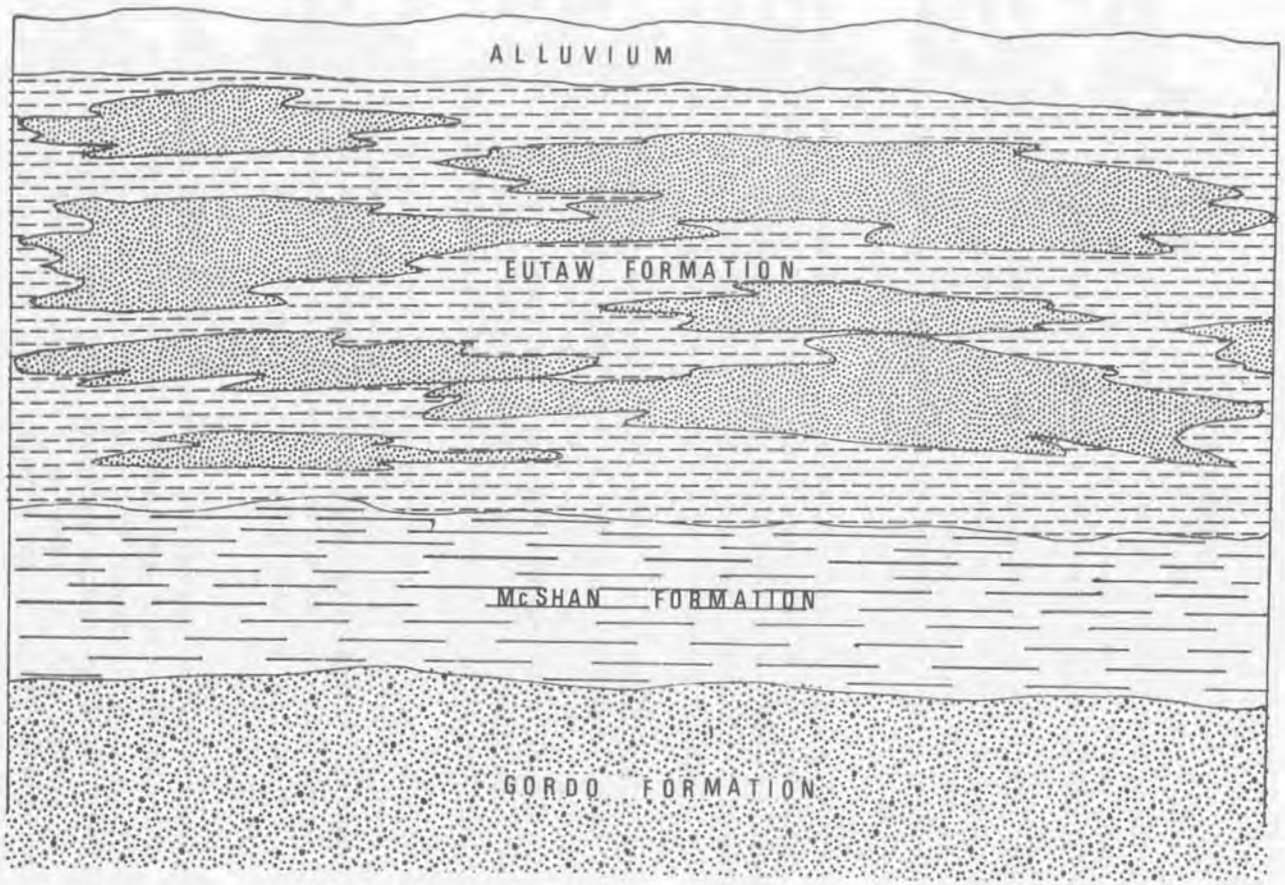

Figure 1.-- Generalized geologic cross section of the Divide Section.

Several problems became apparent during the dewatering studies. One is that the volume of material to be dewatered is large. As much as 150 ft of saturated material over a 27-mile stretch through the Tennessee Valley divide must be dewatered. Because of the immensity and timing of this project, it is imperative that the most efficient dewatering system be used. Other problems arise from the physical character (low horizontal and vertical hydraulic conductivity) of the material to be dewatered. Two methods of dewatering were studied for this project--by wells and trenching.

FREE-FLOWING WELL ANALYSIS

Pressure relief in the Gordo Formation would involve installing free-flowing wells at regular intervals on both sides of the centerline of the Waterway. These wells would be installed before construction of the Waterway and would then be cut off to discharge at lower elevations as material is excavated around the wells.

The problem of calculating drawdown caused by free-flowing wells is difficult to solve analytically when more than one well is involved. The problem is further complicated by the progressive lowering of the discharge elevations at various points in time. It was determined that the problem could be solved by digital simulation.

A two-dimensional finite difference ground-water flow model developed by Trescott, Pinder, and Larson (1976) was used for this study. Modifications were made to the model to simulate this flow problem. For confined aquifers, the model solves the two-dimensional ground-water flow equation:

$$T_{xx} \frac{\partial^2 h}{\partial x^2} + T_{yy} \frac{\partial^2 h}{\partial y^2} = S \frac{\partial h}{\partial t} + W(x,y,t) \qquad (1)$$

where T_{xx}, and T_{yy} are components of the transmissivity tensor, h is the hydraulic head, S is the storage coefficient, t is time, and $W(x,y,t)$ is the volume of recharge or withdrawal per unit surface area of the aquifer per unit time. It is assumed the Cartesian coordinate axes x and y are aligned with the major components of the transmissivity tensor T_{xx} and T_{yy}.

In a model such as this one, the area to be simulated is divided into rectangular subareas with a variable-width, finite-difference grid. Within each subarea or cell the hydraulic properties and stresses are assumed to be constant. A finite difference approximation of equation 1 is applied to the grid system and the resulting set of simultaneous equations are solved by one of several iterative techniques. Before a grid can be set up, however, the proper representation for the wells and the boundaries must be established.

If the wells on either side of the centerline are adjacent and the lines of wells can be considered straight, parallel, and infinitely long, and the aquifer characteristics do not change in the direction of the lines, then symmetry can be used to reduce the size of the model. Lines of no flow (lines of symmetry) perpendicular to the centerline of the trench exist (1) through adjacent wells and (2) midway between adjacent wells (fig. 2). The centerline of the trench is a third no-flow line.

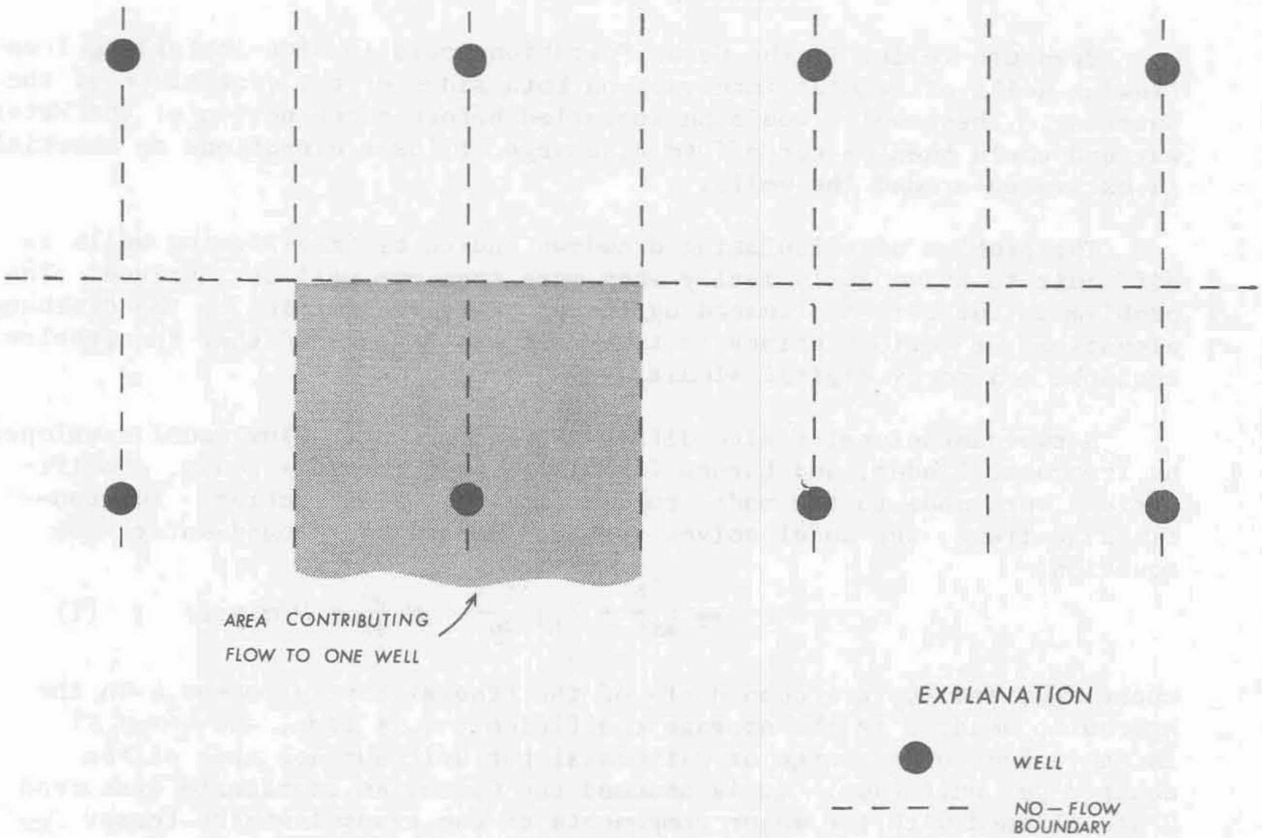

Figure 2.-- No-flow boundaries established by two infinitely long parallel lines of wells.

Using these no-flow boundaries only one-half of a free-flowing well and the area contributing to it's flow need be simulated.

With a generalized model such as this one the fourth boundary can be established as a constant-head or no-flow boundary placed at such a distance away that it will have no effect on the solution in the area of interest. If a real boundary is known to exist near the lines of wells then it could be represented as such in the model. In doing so, however, the problem may not be symmetrical about the centerline and it will be necessary to include in the finite difference grid the one-half well and the appropriate boundary on the other side of the centerline. In the area of this study, aquifer boundaries are distant from the lines of wells and representing them several ways did not affect the solution in the area of interest. For these simulations the grid was established by starting with the cell representing the flowing well and expanding the dimensions of the rows and columns by a factor of about 1.5 until the proper distances were represented (fig. 3).

Simulated flow into a constant head cell is a function of the size of the cell, just as flow into a real well is a function of the radius of the well; therefore it is necessary to determine the cell dimensions that will approximate flow to a real constant-head well. An analysis done by Prickett (1967) showed that effective well radius can be related to the grid size of a square cell by

$$r_w = a/4.810 \qquad (2)$$

where a is one dimension of the cell and r_w is the effective radius of the well. This relationship cannot be used, however, for the dimension of the cell that is perpendicular to the no-flow boundary (fig. 4). The same analysis done for this dimension of the cell indicates that the relationship is

$$r_w = b/2.193 \qquad (3)$$

where b is the dimension of the cell that simulates water flowing into $\pi/4$ radians of the well adjacent to the no-flow boundary (fig. 4). Using the dimensions $a = 4.810\ r_w$ and $b = 2.193\ r_w$, the size of the cell needed to simulate the flowing well is defined.

Problems from uplift pressure will occur only when enough material is removed from upper layers to establish a sufficient pressure differential between the upper and lower surfaces of the confining layer to cause breaching of the confining bed. It was determined by the Corps of Engineers that during the construction process the pressure due to the materials above the confining layer should be at least 1.5 times the artesian pressure on the base of the confining layer. If two parallel lines of flowing wells are used, the point of maximum artesian pressure between the lines of wells occurs at the center of the rectangle formed by four adjacent wells.

To determine the approximate maximum rate that excavation could take place for a particular well spacing and given sequence of lowering the well discharge level, an algorithm was written into the code of the model to compute the upward and downward force on the confining layer after each time step at the point of maximum pressure. When the upward force at

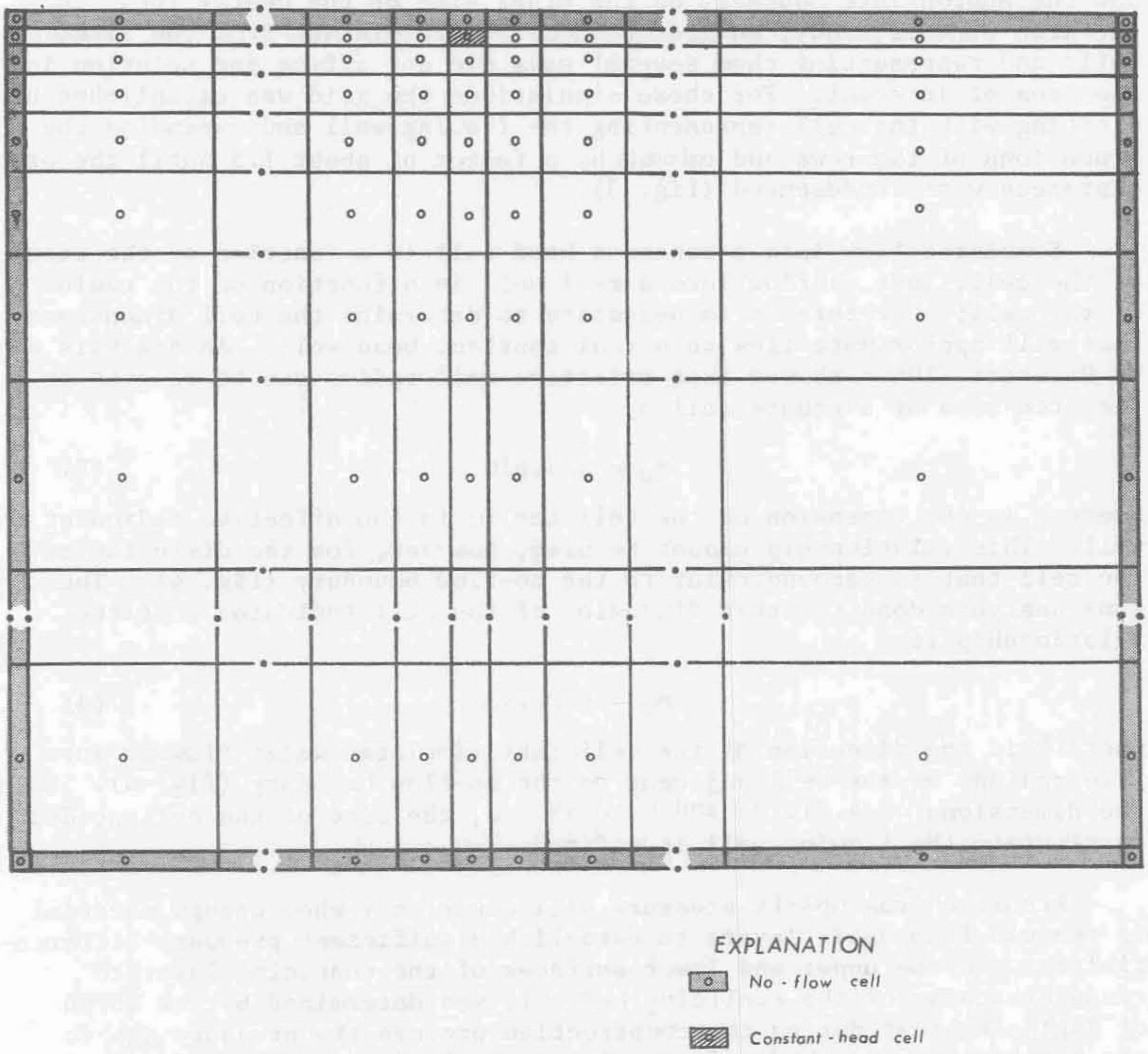

Figure 3.-- Finite difference grid used in simulation of two parallel lines of free-flowing wells.

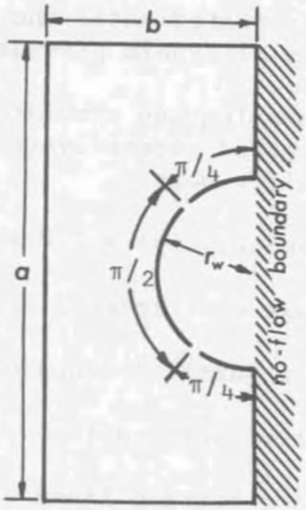

Figure 4.-- Relationship of cell dimensions a and b to effective well radius r_w for simulation of flow to one half of a well.

the point declined to a value less than 1.5 times the downward force for excavation at the next lower discharge level, the head in the well was instantaneously lowered to the next interval and a new "pumping period" was started. The process was repeated until the pressure was lowered to a value less than the maximum allowable pressure for the complete excavation.

To test the model a problem was simulated involving typical geologic conditions with the anticipated stress system. The uniform hydraulic characteristics of the aquifer were:

T (Transmissivity) = 230 ft^2/day and

S (Storage coefficient) = 0.0002

stressed by a system of two parallel lines of free-flowing wells with

effective well radius = 0.42 ft

distance between lines of wells = 475 ft and

distance between wells in a line= 2000 ft

In addition to these conditions, the wells discharge initially at 24 ft below the original potentiometric surface and were later lowered to discharge at distances of 34 ft, 44 ft, 54 ft, and 64 ft below the original potentiometric surface. The discharge elevation was lowered 10 ft when the safety factor criterion for excavation to the next lower interval was met.

The pressure at the cell closest to the point of minimum drawdown can be plotted to observe how head reduction is taking place with time (fig. 5). It is also useful to observe the change in discharge flowing into each well with time (fig. 6). With this information it is possible to predict the maximum rate of excavation that could take place with least danger of a blowout in the confining layer.

SIMULATION OF DEWATERING BY TRENCHING

Calculations of the effects of flow to a trench can be made using published analytical or numerical solutions that consider the problem of flow to a fully penetrating trench in an isotropic aquifer. One of the most widely used solutions is derived by linearizing and solving the one-dimensional unconfined ground-water flow equation

$$K \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) = S_y \frac{\partial h}{\partial t} \qquad (4)$$

subject to the boundary conditions (Yeh, 1970),

When $t = 0,\ h = h_o$, for $x \geq 0$,

for $x = 0,\ h = d$, when $t > 0$,

and for $x = \infty,\ h = h_o$, when $t \geq 0$,

where d, h, h_o, and x are dimensions indicated on figure 7, t = time,

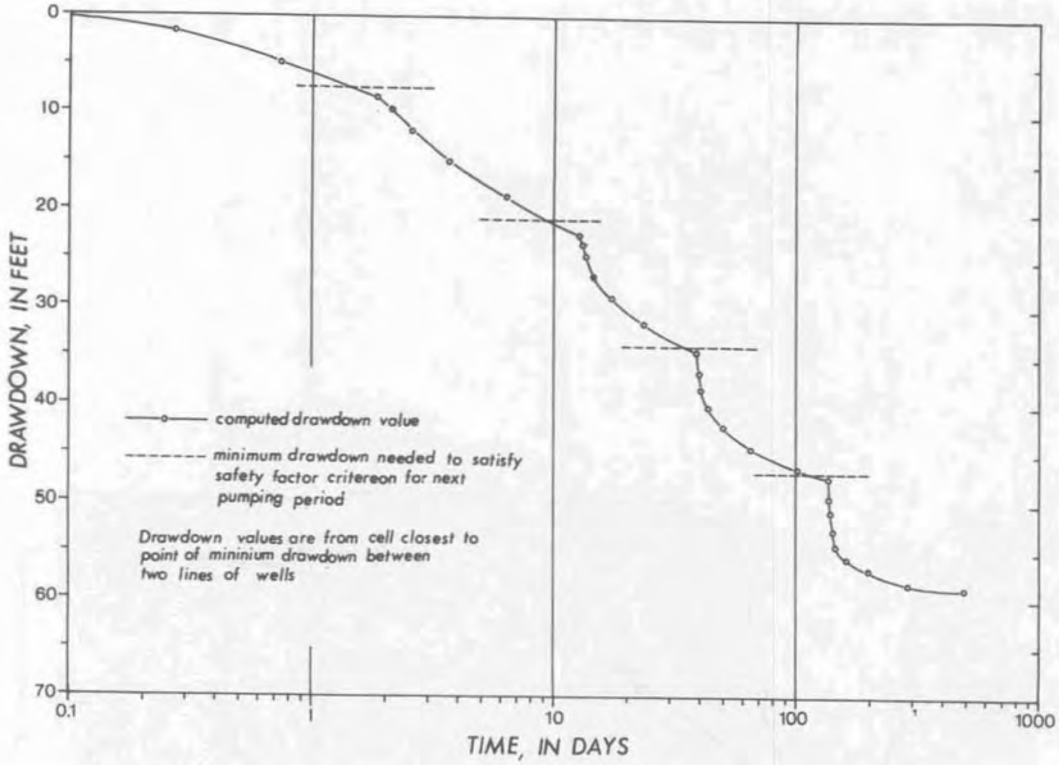

Figure 5.-- Time-drawdown relationship for sample problem.

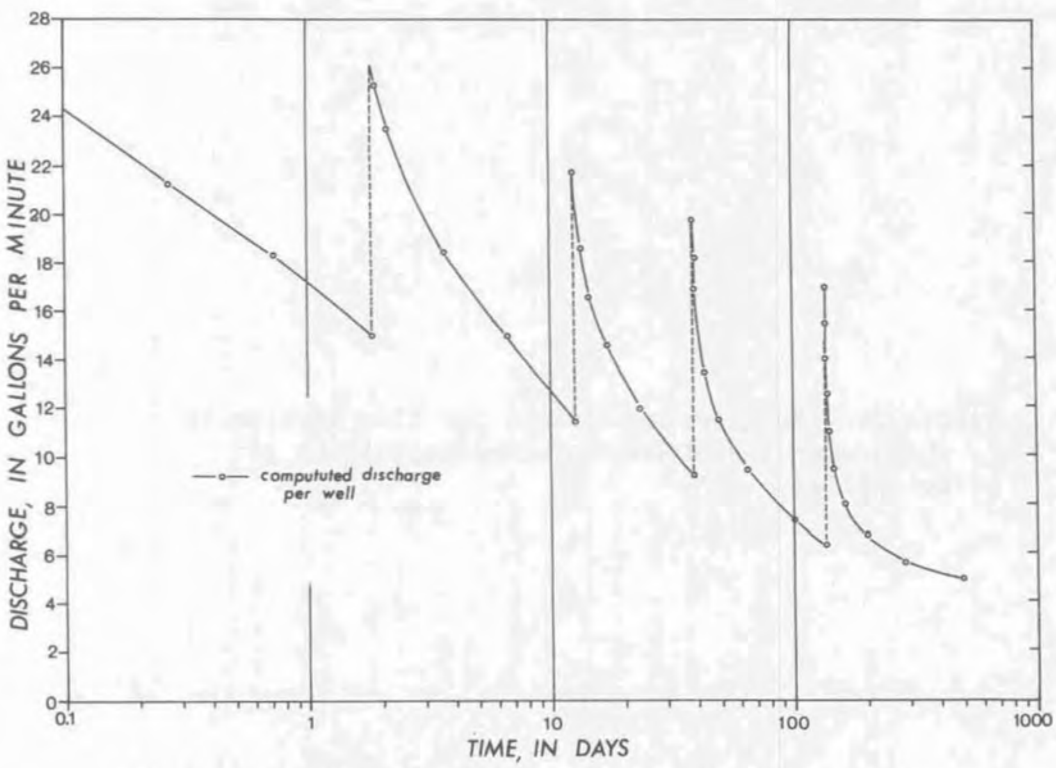

Figure 6.-- Time-discharge relationship for sample problem.

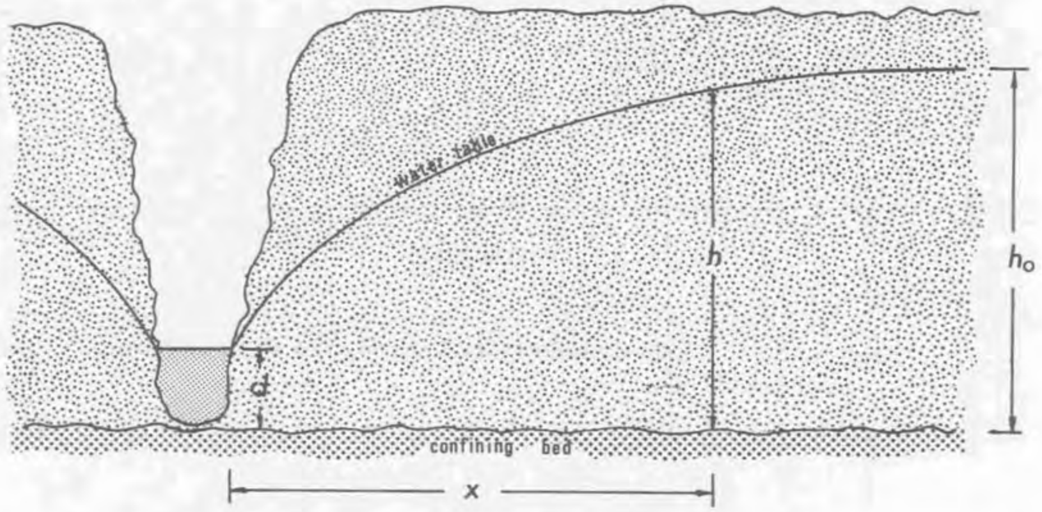

Figure 7.-- Boundary conditions for flow problem to which one-dimensional solution techniques can be applied.

K = hydraulic conductivity, and S_y = specific yield of the aquifer. The resultant solution is:

$$h = d + (h_o-d)\operatorname{erf}(x/\sqrt{4h_oKt/S_y}\ ) \qquad (5)$$

The error function, erf (x), is given in tables, and is a standard function in many computer libraries. This solution is reliable when d is large compared to the drawdown in the trench (h_o-d). For situations where this is not the case, an additional solution developed by Yeh (1970) is applicable. This solution was developed by solving equation 4 with numerical techniques. The solution is given in a table as values of h/h_o for values of $\phi = x/\sqrt{h_oKt/S_y}$ and d/h_o.

If the aquifer is anisotropic and the trench does not fully penetrate the aquifer, the two- or three-dimensional ground-water flow equations must be solved. To study the effects of dewatering by trenching in the Divide Cut Section, a two-dimensional digital model was applied to a cross section of the area.

The Trescott, Pinder, Larson (1976) finite difference model used in the free-flowing well analysis was modified to simulate flow to a trench with a moveable upper boundary representing the water table. If the cross section is taken as a uniform width (b), then equation 1 can be re-written and applied to the cross section in the form

$$K_{xx}b\,\frac{\partial^2 h}{\partial x^2} + K_{yy}b\,\frac{\partial^2 h}{\partial y^2} = S_s b\,\frac{\partial h}{\partial t} \qquad (6)$$

with the assumptions that K_{xx} and K_{yy}, the principal components of the hydraulic conductivity tensor, are not functions of x and y, respectively. For cells in the finite-difference grid simulating the water table (fig. 8), the specific yield, $S_y/\Delta y$, is used in the right-hand side of the equation in place of S_s, where Δy is the y dimension of the cell.

To move the boundary concurrently with head reduction, the mechanics of the model were modified to move the upper boundary by dropping out cells at the water table as the head declined to a value less than the distance from the base of the aquifer to the center of the cell. The specific yield would then be used in the equation for the next lower cell in a column. With this arrangement, the upper boundary of the modified model moves with the decline of the water table (fig. 8).

Constant-head cells were used to simulate a trench. The trenching model has the capability of adding constant-head cells or changing constant-head values at selected points in time. With this arrangement, it is possible to simulate the change in the water level in a trench, the deepening of a trench, or the addition of a new trench at desired points in time.

To test the cross-sectional model, an idealized problem that could be solved by other methods was simulated. The problem included a trench fully penetrating a 100-foot thick isotropic aquifer with a specific yield of 0.2, and a hydraulic conductivity of 15 ft/day. With an

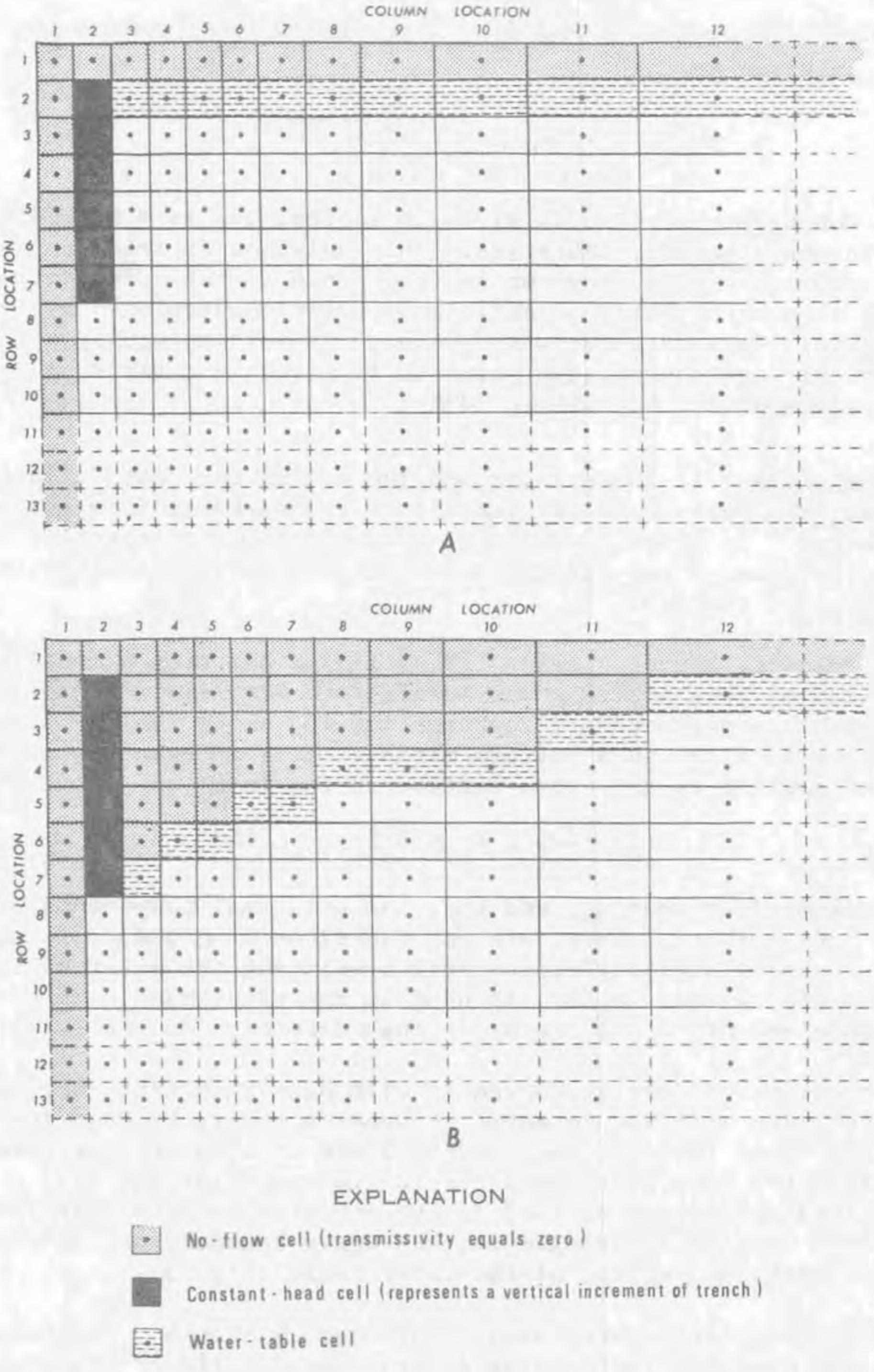

Figure 8.-- Finite-difference grid used in moveable-boundary cross-sectional model simulating the effects of a partially penetrating trench (from Leake, 1977).

assumed instantaneous head decline of 80 ft in the trench, water-surface profiles were calculated for different times. For comparison purposes, this problem was solved using the cross-sectional model, a one-dimensional finite-difference model, and by Yeh's method. Water-surface profiles from the cross-sectional model and the one-dimensional model indicate agreement to within about 1 ft or 1 percent of the initial aquifer thickness. Values from Yeh's solution show slightly more drawdown than do the finite difference models (fig. 9). The agreement, however, is still good.

One proposed method of trenching is to construct a trench that penetrates a given percentage of the aquifer then deepen it in increments at selected points in time until the entire aquifer is penetrated and the desired dewatering has occurred. Because the aquifer thickness varies in the divide section, it is desirable to obtain a single solution that, for a given sequence of trenching, is universal with respect to thickness and other aquifer characteristics. To accomplish this, a technique for non-dimensional simulation was developed by Leake (1977). This was done by making non-dimensional variable substitutions for x, y, and h in equation 6 and rearranging the equation so that parameters could be combined with time (t) to form non-dimensional time, t^*. The non-dimensional results can then be interpreted with the original values of x, y, K_{xx}, S_y, and h_o, and the relationship of t to t^*.

A problem involving a single trench penetrating one half of the aquifet thickness and then deepened to penetrate the remaining half of the aquifer was simulated to illustrate the results of the model. For this simulation, time (t) is given by

$$t = \frac{30.S_y h_o t^*}{K_{xx}}$$

where h_o is the original aquifer thickness, S_y is the specific yield of the aquifer, K_{xx} is the horizontal hydraulic conductivity of the aquifer, t^* is non-dimensional time and the constant, 30, results from the use of 30 grid intervals over the vertical domain. The ratio of vertical to horizontal hydraulic conductivity is given as 0.02 for this problem. The trench was assumed to be instantaneously made at time zero and deepened at non-dimensional time $t^*= 30.5$. The results can be represented graphically with water-surface profiles in terms of percent of aquifer thickness dewatered and non-dimensional distance from trench. A family of curves for various non-dimensional times is shown in figure 10.

SIMULATION OF DEWATERING BY PUMPED WELLS

The most widely used systems for major construction dewatering involve pumping one or more wells proximate to the area to be dewatered until the water table has been lowered to the desired level. For a project comparable with the divide cut section of the Tennessee-Tombigbee Waterway, the dewatering system would most likely include one or more lines of evenly spaced dewatering wells parallel to the excavation. To test the effectiveness of this method, the Nashville District of the U.S. Army Corps of Engineers designed and constructed a dewatering system that included more than 40 wells in two parallel lines on either side of the proposed waterway. The wells, spaced 500 ft apart in a line, were constructed with screens open to sands in the Eutaw Formation, McShan

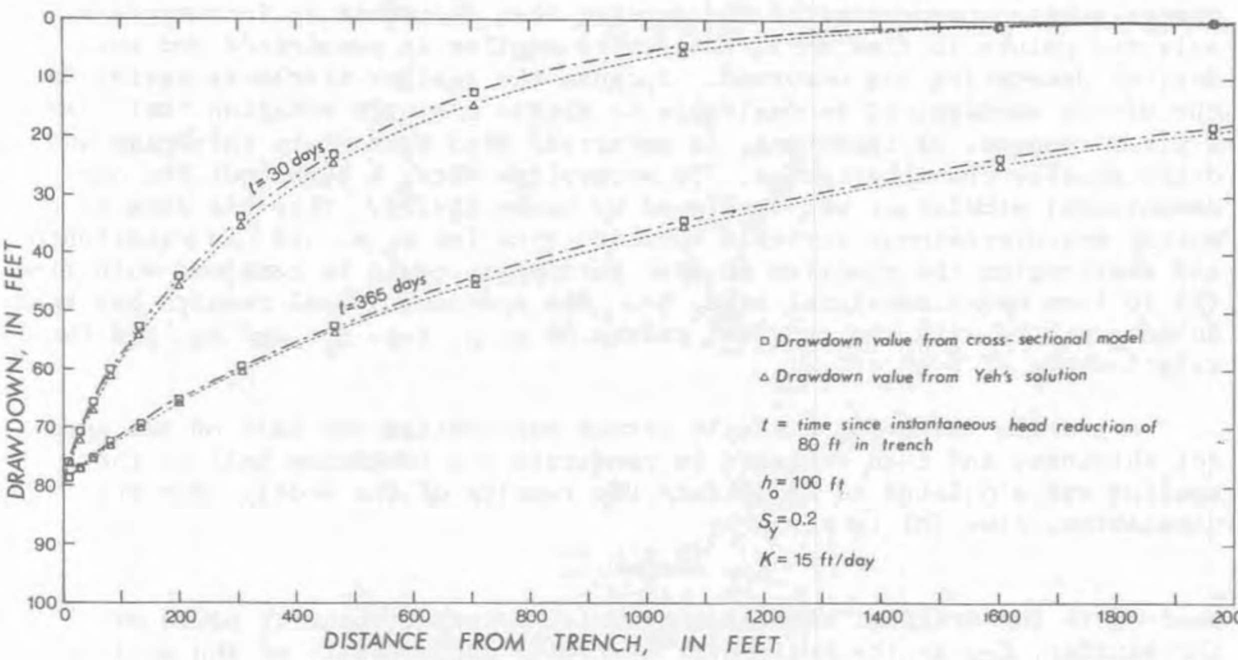

Figure 9.-- Comparison of results from cross-sectional model with results from one-dimensional solution (modified from Leake, 1977).

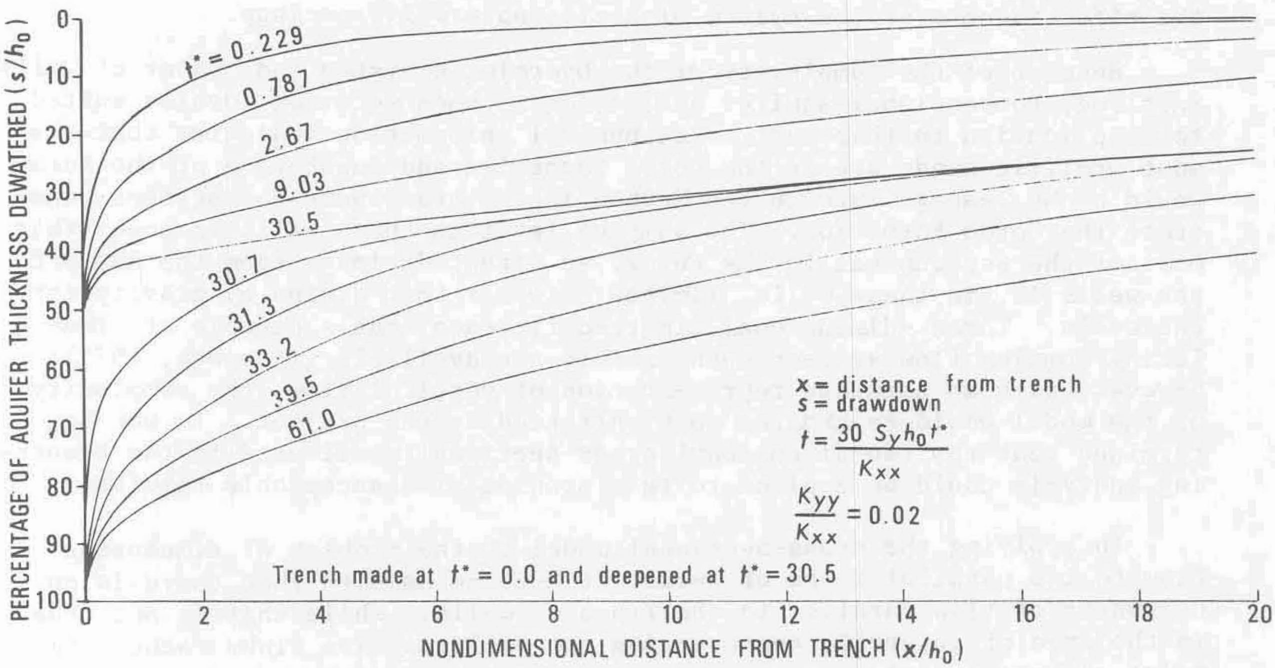

Figure 10.-- Nondimensional water-surface profiles from sample problem (from Leake, 1977).

Formation, and the upper part of the Gordo Formation in the northern part of the test section and with screens open only to the Eutaw and McShan in the southern part of the section. The average discharge for the wells was about 50 gal/min; however, individual well production ranged from less than 10 gal/min to about 300 gal/min. Clusters of piezometers having a wide vertical distribution of screen openings were placed at several locations between the lines of wells to monitor the potentiometric surface in the area to be dewatered. The objectives of the study were to implement the dewatering system, evaluate it's effectiveness, use the results to determine how long the system would take to complete the desired dewatering, and determine the effectiveness of the system with alternate well spacings.

Because of the complexity of the hydrologic system and number of wells involved, conventional aquifer evaluation methods were not ideally suited for application to this test. Geophysical information indicates that the most prolific sands are in the Gordo Formation and dewatering of the Eutaw would be by leakage through the McShan in the area where the screens penetrate the Gordo Formation. The pumping level in these wells is lower than most of the screens set in the Eutaw, so direct drainage from the Eutaw to the wells is, in these wells, limited to water that drains by gravity into the wells. Three -dimensional finite-difference models capable of simulating complex flow systems such as this are available (Trescott, 1975); however, with an accurate representation of vertical flow, the complexity of the model would be greater than warranted by the project. It was determined that the two-dimensional cross-sectional model used in the trenching analysis could be applied to this problem with acceptable results.

In applying the cross-sectional model to the problem of simulating flow to two parallel lines of wells, it must be assumed that there is no component of flow parallel to the lines of wells. While this is not true in the area of interest, representing the wells as line sinks rather than point sinks can yield a reasonable approximation of the potentiometric surface between the lines of wells if the discharge for a well is evenly distributed over the length of the line sink representing the well (fig. 11). Intuitively, this approximation would result in a potentiometric surface that would have about the average values from profiles taken parallel to the lines of wells. In this study, the wells are close together and the potentiometric surface between the wells is relatively flat.

The results from the dewatering test show a very slow drainage from the alluvium. Hydrographs for piezometers open to the Eutaw, McShan, and Gordo indicate a rapid potentiometric decline of 20-30 ft during the first 10 days of the test followed by a much slower decline for the next 200 days. The change in rate-of-decline indicates that a change from confined to unconfined conditions is taking place. To simulate this in cross section, it was necessary to modify the cross-sectional model. In a conversion problem, the storage term in the finite difference equation must include the storage coefficient, S, when the head is above the top of the aquifer, and the specific yield, S_y, when the head is below the top of the aquifer. In addition, when the aquifer goes from confined to unconfined during a time step that part of the head reduction below the top of the aquifer should be a function of the specific yield and that part of the head reduction above the top of the aquifer should be a function of the storage

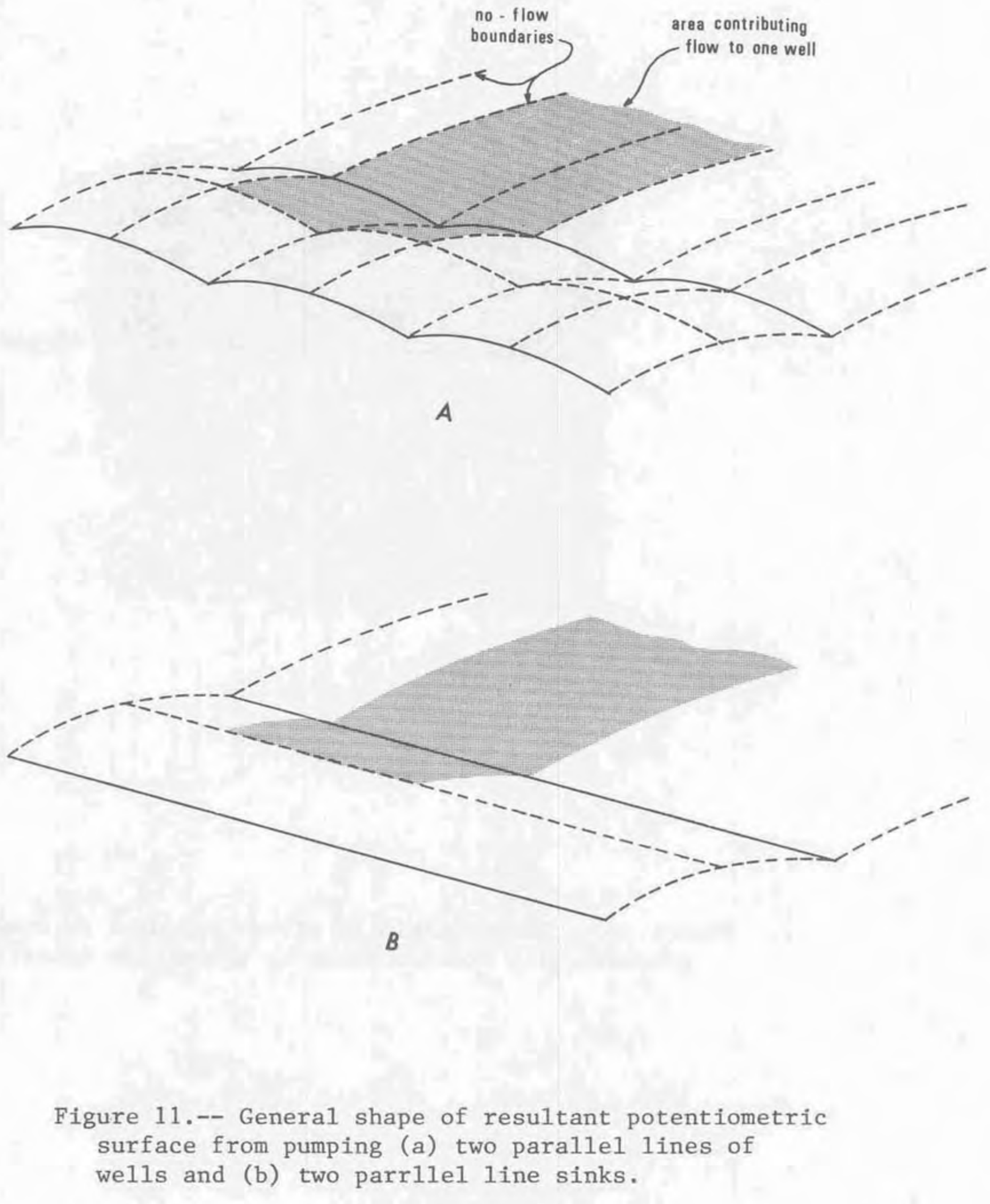

Figure 11.-- General shape of resultant potentiometric surface from pumping (a) two parallel lines of wells and (b) two parrllel line sinks.

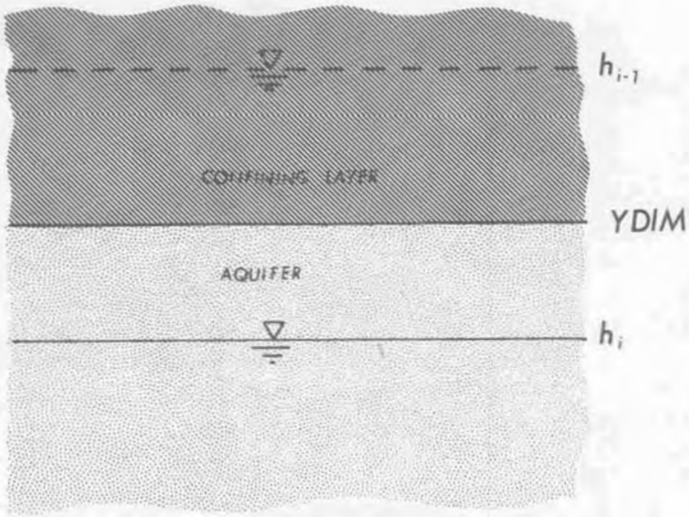

Figure 12.-- Relationship of elevations used in simulation of conversion from confined to unconfined conditions.

coefficient. To simulate this a term

$$SUBS = (h_{i-1} - YDIM)\ (S - S_y)/\Delta t$$

is added to the storage term

$$S_y(h_i - h_{i-1})/\Delta t$$

if a change from confined to unconfined conditions occurs during the time step, Δt. The variable h_i, h_{i-1}, and YDIM are elevations shown in figure 12. The result is that the incorrect part of the storage term is subtracted and a correct part is added (Trescott, Pinder, and Larson, 1976).

To apply the model to the dewatering test, a finite difference grid was established representing a cross section perpendicular to the lines of wells. The cross section was constructed from the centerline between the two lines of wells to more than 10 mi from the centerline. The section had a uniform thickness of 340 ft. The upper 90 ft represents the Eutaw and McShan Formations and the lower 250 ft represents the Gordo Formation.

Values of horizontal hydraulic conductivity (K_{xx}) were 7.5 ft/day for the Eutaw-McShan system and 30 ft/day for the Gordo Formation. The ratio of vertical to horizontal conductivity (K_{yy}/K_{xx}) used was 0.01. An initial head of 360 ft was used for the entire area with a storage coefficient of 0.00001 and a specific yield of 0.1 applied to the confined and unconfined areas, respectively.

The discharge was 50 gal/min divided by the well spacing of 500 ft. In these simulations all discharge was assumed to come from the upper 20 ft of the Gordo Formation and was distributed uniformly within that vertical interval. Leakage from the alluvium to the Eutaw and drainage from the Eutaw into the wells was not considered.

The area of interest for dewatering is along the centerline between the two lines of wells. Most of the piezometers are located along this line. The head values from cells corresponding to the location of piezometers can be compared with the piezometer readings obtained from pumping test to determine the ability of the model to simulate flow in the system. A comparison for the first 100 days of the test is given in fig. 13. The model could now be used to predict responses to alternate well configurations and long-term simulations.

SUMMARY

Ground-water flow models, although primarily considered a water-management tool, can be applied to hydraulics problems related to construction dewatering. For a study done with the U.S. Army Corps of Engineers, Nashville District, finite difference modeling techniques were used to obtain solutions to dewatering problems.

The effects of two straight, parallel, and infinitely long lines of free-flowing wells were determined by simulating a part of the flow system in plan view. No-flow boundaries due to symmetry were used to reduce the area of the system that was simulated. The proper cell dimensions

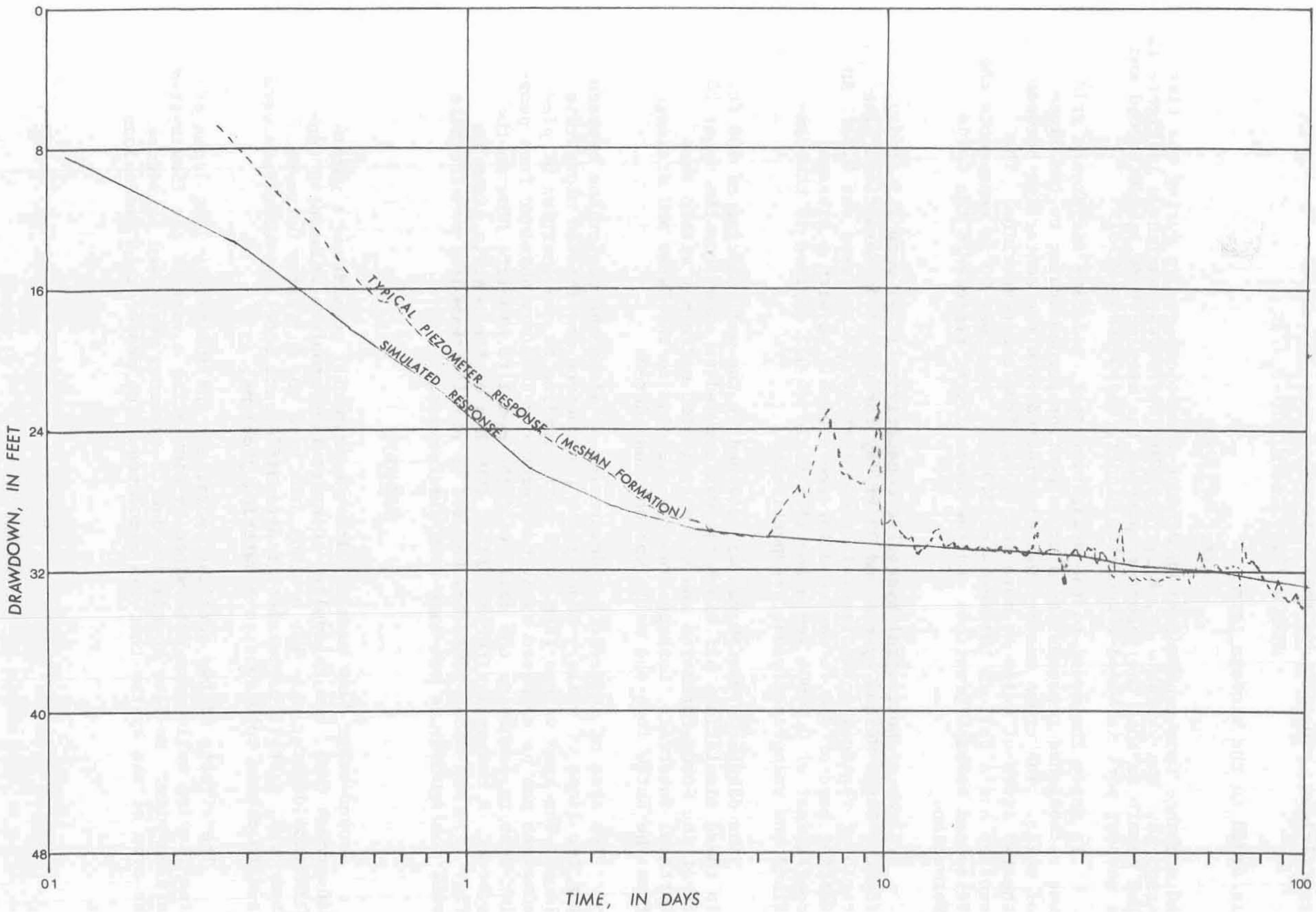

Figure 13.-- Comparison of simulated and observed drawdown from pumping two parallel lines of wells.

representing a constant-head well were established and the model was modified to simulate a pressure reduction schedule that could occur during excavation of the Tennessee-Tombigbee Waterway.

To determine the effects of dewatering by trenching, a two-dimensional finite-difference model was applied to a cross section of the area. The model was modified to move the upper boundary to correspond with the water-level decline. At selected points in simulated time, constant-head nodes were added to simulate the deepening of a trench. To apply the results to a variety of conditions in the area to be dewatered, a non-dimensional simulation technique was used.

The third problem handled with a finite-difference model was the simulation of flow to two lines of constant-discharge wells. With the assumptions that the lines were straight and infinitely long, and that discharge was distributed evenly over the spacing of the wells in a line, the moveable-boundary cross-sectional model was applied. Because results of a dewatering test indicated that a change from confined to unconfined conditions as taking place, it was necessary to modify the model to simulate this conversion. The model results were in good agreement with piezometer responses.

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